References

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Comment on "Interception of High-Speed Target by Beam Rider Missile"

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IN Ref 1, it is possible to express the missile coordinates explicitly in terms of the normal elliptic integrals F and E

The linear differential equation considered by the authors was

$$(d/d\gamma)(\cot\theta) + \tfrac{1}{2}\cot\gamma\cot\theta + \tfrac{1}{2} = 0 \qquad (1)$$

Let ds_m be an element of arc of the missile trajectory and

$$ds_m = V_m dt = (V_m/V_t)V_t dt = \tau R d(\cot\theta)$$

With $\tau R = c$ and $\cot\theta = x_m/y_m$, we have,

$$ds_m = cd\left(\frac{x_m}{y_m}\right) = c\left(\frac{dx_m}{y_m} - \frac{x_m dy_m}{y_m{}^2}\right) \qquad (2)$$

Since

$$dx_m = \cos\gamma ds_m \qquad dy_m = \sin\gamma\, ds_m$$

Eq (2) becomes

$$ds_m = c\left(\frac{\cos\gamma}{y_n} - \frac{x_m \sin\gamma}{y_m{}^2}\right) ds_m \qquad (3)$$

Therefore,

$$y_m{}^2 = c(y_m \cos\gamma - x_m \sin\gamma) \qquad (4)$$

This equation clearly shows that the velocity of the missile is tangent to the circle centered at the origin and of radius $y_m{}^2/c$, a result mentioned in Ref 2 using different arguments

Therefore,

$$\cot\theta = \frac{x_m}{y_m} = \cot\gamma - \frac{y_m}{c\sin\gamma} \qquad (5)$$

Using Eq (5) in (1), one obtains the differential equation

$$2\sin\gamma(dy/d\gamma) - \cos\gamma y + 1 = 0 \qquad (6)$$

which is also linear with $y = y_m/c$

Integrating Eq (6) yields

$$y = \cos\phi\left(C + \frac{F(\phi,k) - 2E(\phi,k)}{2^{1/2}}\right) + \sin\phi(1 + \cos^2\phi)^{1/2} \qquad (7)$$

and using (4) we obtain the x coordinate as

$$x = -\left(C + \frac{F - 2E}{2^{1/2}}\right) \times \left[C + \frac{F - 2E}{2^{1/2}} + \tan\phi(1 + \cos^2\phi)^{1/2}\right] \qquad (8)$$

where F and E are normal elliptic integrals of the first and second kind with moduli $k = 1/2^{1/2}$ and arguments $\phi = \text{arc}\cos(\sin\gamma)^{1/2}$

The constant C is determined by the initial conditions We also have

$$\cot\theta = \frac{x}{y} = -\frac{1}{\cos\phi}\left(C + \frac{F - 2E}{2^{1/2}}\right) \qquad (9)$$

which is Eq (9) in Ref 1

Using the initial conditions, when $x = y = 0$, $\theta = \theta_0 = \gamma_0$, we have for the constant C,

$$C = \frac{2E(\phi_0) - F(\phi_0)}{2^{1/2}} - \tan\phi_0(1 + \cos^2\phi_0)^{1/2} \qquad (10)$$

By these expressions it can easily be seen that:

1) $y = 1$ is an asymptote since x becomes infinite for $\gamma = 0$ except when

$$C - \frac{2E(\pi/2) - F(\pi/2)}{2^{1/2}} = 0$$

or $C = 0\ 599$ This gives $\theta_0 = 37°$

2) The points where the velocity is directed vertically are such that $\gamma = \pi/2$, $\phi = 0$ Therefore $x = -C^2$ and $y = C$

Hence, they are situated on the parabola $y^2 = -x$, a result also mentioned in Ref 2 using different arguments

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Received September 18, 1963

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"Equilibrium" Gas Composition Computation with Constraints

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IT has been suggested in a recent note[1] that one may "freeze" a particular species in a chemical equilibrium computation by introducing a fictitious second set of species and manipulation of the real and imaginary sets of species, within the usual procedures of the computation routine The purpose of this note is to point out that such a constraint may be imposed on the equilibrium calculation in another manner when one is using the popular minimization of free energy technique[2] In this approach the Gibbs free energy is minimized subject to the constraints of the mass balances

$$\sum_{i=1}^{n} a_{ji}x_i = b_j \qquad (1)$$

where a_{ji} is the number of atoms of element j in species i, x_i is the moles of species i per unit mass, and b_j is the number of gram-atoms of element j per unit mass

It is possible, however, to impose *a priori* relations (constraints) among the x_i composition variables by considering the constraints to be pseudo elements, as long as the relations are of the form of Eq (1), i e, linear Thus the formula matrix a_{ji} would have a column for each species and m rows for each of the m real chemical elements, as usual, with an additional row for each constraint Possible constraints

Received October 11, 1963 This work was carried out under Contract No DA-01-021 ORD-11878(Z) Mod No 7

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